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Effects of Time Delay on Three Interacting Species System with Noise

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Abstract We study the effects of time delay in three interacting species system with noise. The time evolution and spatiotemporal pattern in the Lotka-Volterra model of three interacting species with noise and time delay were investigated by means of stochastic simulation. Our results indicate that: (i) Time delay induces the synchronously periodic oscillations of the three species densities; (ii) Time delay cause the spatiotemporal pattern to be concentrated.

Keywords Interacting species system · Time delay · Noise · Stochastical simulation

1 Introduction

The Lotka-Volterra model, originally introduced by Vito Volterra for description of struggle for existence among species [1–3], has been paid considerable attention in the fields of medicine [4], biology [5], ecology [6–8], and mathematics [9, 10] etc. It helps us understand the diversity of species in the process of biological evolution, where environmental noise plays a beneficial role [11]. The noise, through its interaction with the nonlinear ecosystems, can cause counter-intuitive phenomena such as stochastic resonance, noise-delayed extinction, and spatial patterns [12, 13] etc.

However, in realistic systems, consideration of time delay is more natural. In the field of pure statistical physics, the systems with noise and time delay are the most popular objects of research because the combinations of noise and time delay makes them more complex and richer dynamic behaviors [8, 14–18]. In the ecological field, little effort has been devoted to study of the effects of time delay in detail from a statistical view of point. In 1998, José M.G. Vilar and Ricard V. Solé have analyzed the interplay between noise and periodic modulations in a classical Lotka-Volterra model of two-species competition [12]. In 2004, A. Fiasconaro, B. Spagnolo and D. Valenti have investigated the noise-induced pattern formation in a population dynamical model of three interacting species in the coexistence regime [19]. We have

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studied the effects of the time delay on the mutualism system and found that combination of the noise and the time delay completely suppress the population explosion of the mutualism system [20]. In the competitive system of ecological field, but the effects of time delay combination with noise has not been fully understood yet.

In this paper we will go on the research work of A. Fiasconaro, B. Spagnolo and D. Valenti [19] and further investigate the inclusion of time delay into the Lotka-Volterra model of three interacting species (one predator, two preys). The paper is structured as follow: in Sect. 2, the basic statistical properties with time delay are provided; in Sect. 3, the behaviors of spatiotemporal pattern are presented; in Sect. 4, conclusions are made.

2 The Effects of Time Delay on the Time Evolution of Population Densities

The time evolution of two preys and one predator is obtained within the formalism of the Lotka-Volterra equations in the presence of a multiplicative noise [19]. Here, after introducing a time delay, we obtain the following equations [21]:

$$\frac{dx(t)}{dt} = \mu x(t)(1 - x(t-\tau) - \beta(t)y(t-\tau) - \gamma z(t-\tau)) + x(t)\xi_x(t), \tag{1}$$

$$\frac{dy(t)}{dt} = \mu y(t)(1 - y(t-\tau) - \beta(t)x(t-\tau) - \gamma z(t-\tau)) + y(t)\xi_y(t), \tag{2}$$

$$\frac{dz(t)}{dt} = \mu_z z(t)(-\beta_z + \gamma_z(x(t-\tau) + y(t-\tau))) + z(t)\xi_z(t), \tag{3}$$

where x, y are respectively the population densities of two preys, z denotes the density of predator. $\xi_i(t)$ is Gaussian white noise with zero mean and correlation function $\langle \xi_i(t)\xi_j(t')\rangle = \sigma\delta(t-t')\delta_{ij}$ $(i, j = x, y, \sigma$ denotes the strength of the noise), μ is the growth rate of preys, τ is delay time, γ and γ_z are the interaction parameters between preys and predator, respectively, β represents the interaction parameter between two preys, and given by [19]

$$\beta(t) = 1 + \epsilon + \eta\cos(\omega t), \tag{4}$$

here $\eta = 0.05$, $\omega = 2\pi 10^{-3}$ and $\epsilon = -0.01$. The interaction parameter $\beta(t)$ oscillates around the critical value $\beta_c = 1$ in such a way that the dynamical regime of Lotka-Volterra model for two competing species changes from coexistence of the two preys ($\beta < 1$) to exclusion of one of them ($\beta > 1$) [22].

Analytically the basic statistic properties of x, y and z are rather difficult to be obtained. But the (1)–(3) can be stochastically simulated by means of Euler arithmetic [23]. In order to simulation of the (1)–(3), it is much rational to let $x(t-\tau) = y(t-\tau) = z(t-\tau) = x(0) = y(0) = z(0)$ as $t < \tau$ for the initial values in the condition of time delay. The results of the simulations with a time step ($\Delta t = 0.001$) of their time evolutions at different levels of multiplicative noises and different delay times are plotted in Figs. 1a–c.

Figure 1a shows the time evolution of population densities without time delay, i.e., $\tau = 0$: (1) Without noise, i.e., $\sigma = 0$, the population densities of all species take on slightly periodic change with t; (2) As the noise strength increases, i.e., $\sigma = 10^{-6}$, the densities of preys take on periodically anti-correlated oscillation and the density of predator presents slightly periodic change with a little disorder; (3) As the noise strength further increases, i.e., $\sigma = 10^{-1}$, the periodic evolution of all population densities are destroyed.

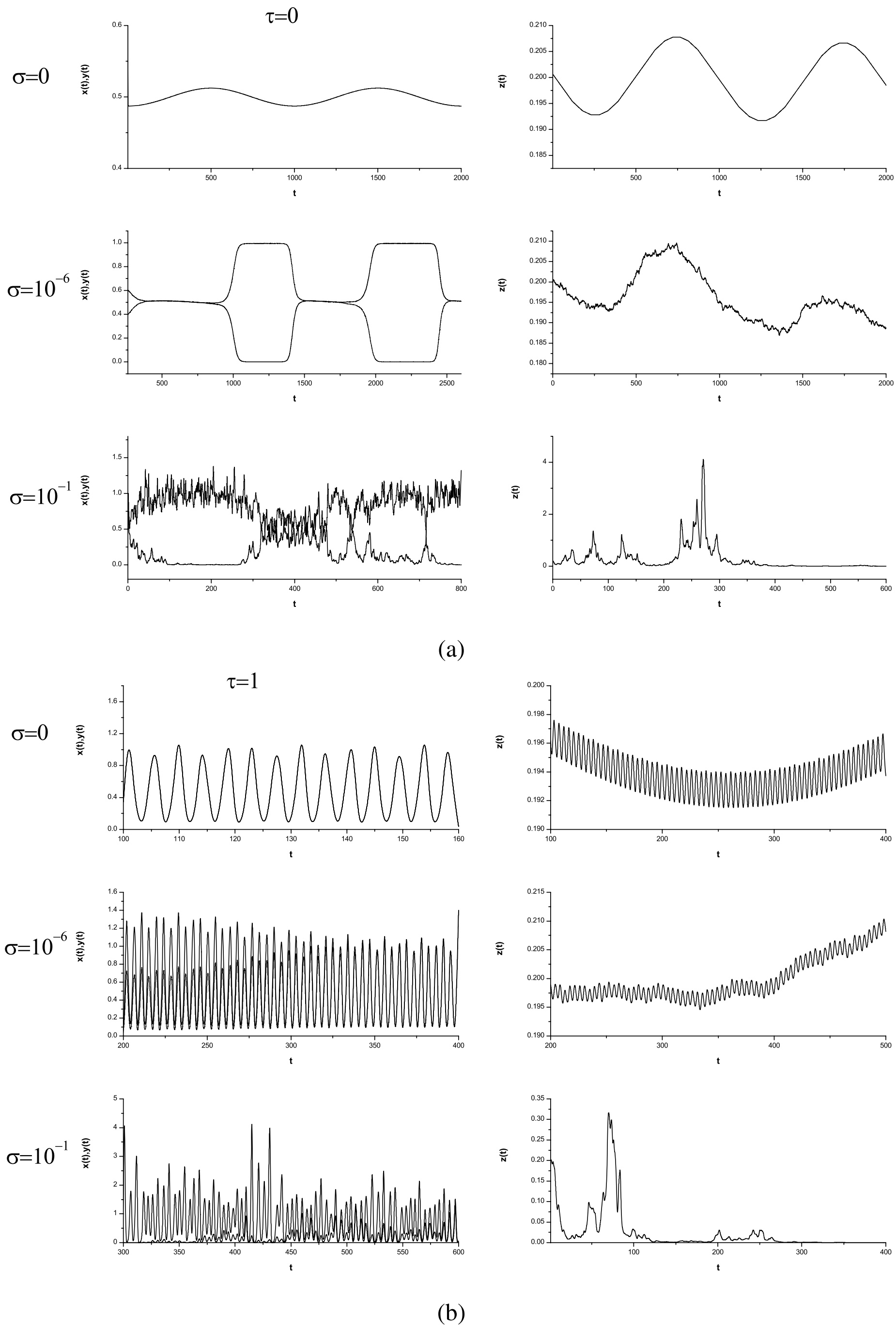
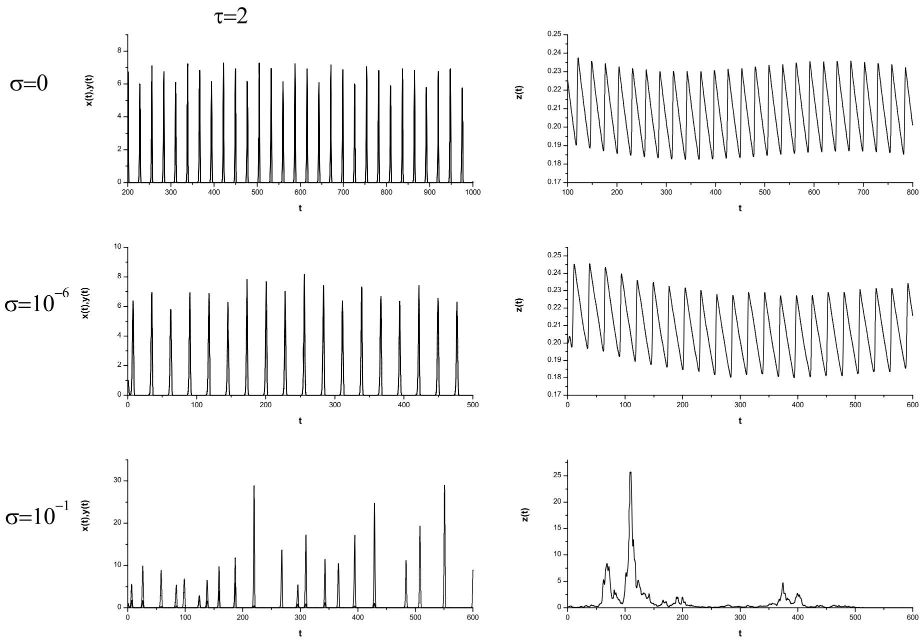


Fig. 1 Time evolution of three populations at different levels of multiplicative noises and different delay times. The values of parameters are $\mu = 2$, $\gamma = 3 \cdot 10^{-2}$, $\mu_z = 1$, $\beta_z = 0.01$, $\gamma_z = 10^{-2}$. The initial values are $x(0) = y(0) = 1$, $z(0) = 0.2$ as $t < \tau$: (**a**) $\tau = 0$, (**b**) $\tau = 1$, (**c**) $\tau = 2$



(c)

Fig. 1 (Continued)

Figure 1b shows the time evolution of population densities with time delay $\tau = 1$: (1) Without noise, i.e., $\sigma = 0$, all species densities present fast periodical oscillations; (2) As the noise strength increases, i.e., $\sigma = 10^{-6}$, the densities of two preys show anti-correlated fast oscillations and the density of predator basically keeps fast periodical oscillation; (3) As the noise strength further increases, i.e., $\sigma = 10^{-1}$, the oscillation of all densities are slightly disorder, but are more regular comparing the case of without time delay.

Figure 1c shows the time evolution of population densities with time delay $\tau = 2$. Figure 1c show that the time series of all species are more regular oscillation than that for the case of short delay time. In a word, the time delay gives rise to the periodically synchronized oscillations of the species densities.

3 The Effects of Time Delay on Spatiotemporal Pattern of Population Densities

Here, the dynamics of our spatially used system is described by the following model of coupled map lattice with time delay (CML) [19, 24, 25]

$$
\begin{aligned}
x_{i,j}^{n+1} &= \mu x_{i,j}^{n}(1 - \nu x_{i,j}^{n-\tau} - \beta^{n} y_{i,j}^{n-\tau} - \alpha z_{i,j}^{n-\tau}) + \sqrt{q}\, x_{i,j}^{n} X_{i,j}^{n} + D\sum_{p}(x_{p}^{n} - x_{i,j}^{n}),\\
y_{i,j}^{n+1} &= \mu y_{i,j}^{n}(1 - \nu y_{i,j}^{n-\tau} - \beta^{n} x_{i,j}^{n-\tau} - \alpha z_{i,j}^{n-\tau}) + \sqrt{q}\, y_{i,j}^{n} Y_{i,j}^{n} + D\sum_{p}(y_{p}^{n} - y_{i,j}^{n}),\\
z_{i,j}^{n+1} &= \mu_{z} z_{i,j}^{n}[-1 + \gamma(x_{i,j}^{n-\tau} + y_{i,j}^{n-\tau})] + \sqrt{q}\, z_{i,j}^{n} Z_{i,j}^{n} + D\sum_{p}(z_{p}^{n} - z_{i,j}^{n}),
\end{aligned}
\tag{5}
$$

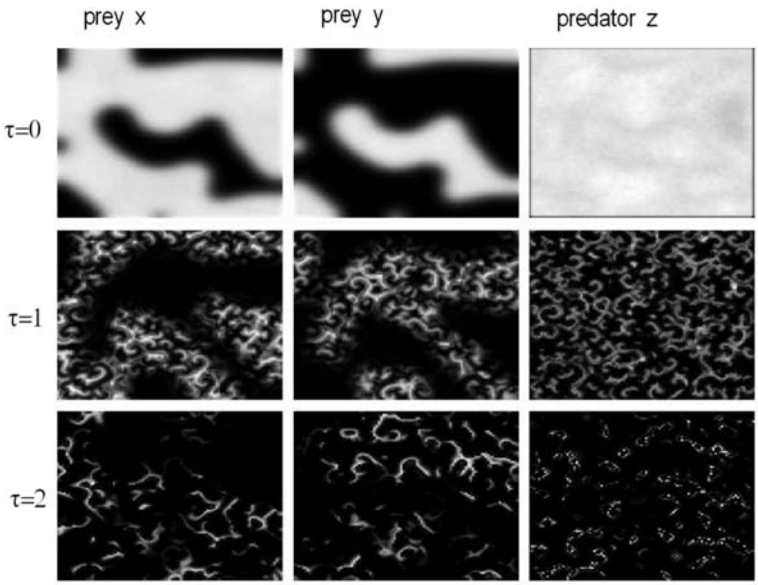


Fig. 2 Spatial pattern formation for preys and predator, at time iteration 600 with $\eta = 0.2$, $\omega = \pi \cdot 10^{-3}$ and $\epsilon = 0.1$. The noise intensity is $q = 10^{-5}$. The other parameters are: $\mu = 2$, $\nu = 1$, $\alpha = 0.03$, $\mu_z = 0.02$, $\gamma = 205$, $D = 0.1$. The initial spatial distribution is homogeneous and equal for all species, i.e. $x_{ij}^{init} = y_{ij}^{init} = z_{ij}^{init} = 0.25$ for all sites (i, j). The *light zones* represent a high density, the *dark ones* represent a low one

where $x_{i,j}^n$, $y_{i,j}^n$ and $z_{i,j}^n$ are respectively the densities of preys x, y and the predator z in the site (i, j) at the time step n, α and γ are the interaction parameters between preys and predator, μ and μ_z are scale factors, τ is delay time, X, Y and Z are the white Gaussian noise variables with

$$\langle X(t)\rangle = \langle Y(t)\rangle = \langle Z(t)\rangle = 0, \tag{6}$$

$$\langle X(t)X(t')\rangle = \langle Y(t)Y(t')\rangle = \langle Z(t)Z(t')\rangle = \delta(t-t'), \tag{7}$$

$$\langle X(t)Y(t')\rangle = \langle X(t)Z(t')\rangle = \langle Y(t)Z(t')\rangle = 0, \tag{8}$$

q is the noise intensity, D is the diffusion coefficient. $\sum_p$ indicates the sum over the four nearest neighbors in the map lattice. The boundary conditions have been established in such a way that no interaction is present out of lattice. This means that for the four corner sites we have only two interactions and for the other 4×98 line-confined sites the number of interactions is three.

For $\beta(t)$, we use the same linear model in Sect. 2

$$\beta(t) = 1 + \epsilon + \eta\cos(\omega t), \tag{9}$$

where, $\eta = 0.2$, $\omega = \pi 10^{-3}$ and $\epsilon = 0.1$. The parameters used in our simulations are $\mu = 2$, $\nu = 1$, $\alpha = 0.03$, $\mu_z = 0.02$, $\gamma = 205$, $D = 0.1$, $q = 10^{-5}$. With this choice of parameters the interspecies competition among the two prey populations is stronger compared to the intraspecies competition (preys-predator), and, therefore, both prey populations can stably coexist in the presence of the predator [22]. The results of simulation are plotted on Fig. 2.

From Fig. 2, we can see that in the absence of time delay, the preys are highly anti-correlated at time steps 600, namely, the two species tend to occupy different positions. But the predator doesn't have obvious pattern. The population distributions of species get concentrated when delay time τ increases. When $\tau = 2$, the distribution of predator represents δ-distribution, moreover, predator and preys are highly anti-correlated.

4 Conclusion

So far, by means of stochastic simulations, we have studied the time evolution and spatiotemporal pattern in the model of three interacting species with noise and time delay. The results indicate that the time delay makes the noise-induced predator-preys system exhibit many peculiar characteristics. The time delay drives the species densities to synchronously periodically oscillate with time. The population distributions of species are concentrated as delay time τ increases. In realistic ecosystem, the distribution of species are always concentrated, therefore, including time delay, we can interpret realistic population dynamics better.

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